Compute Market

Stochastic term structures, portfolio valuation and control

Mathematical specification

Contents

1 Market and balance sheet

Compute Market represents the market as a coupled stochastic term structure. A small set of shared factors reconstructs compute benchmarks, GPU and regional basis, electricity, interest rates and commercial funding spreads at every delivery date. Contracts are functionals of those curves. Physical capacity, debt and cash form the controlled balance sheet.

The complete state is

$$S_t = (X_t, Z_t^{liq}, K_t, \mathcal{B}_t, b_t, D_t, M_t, \theta_{t-}, H_t^{cp}),$$

where X_t contains market factors; Z_t^{liq} is the execution-liquidity state; H_t^{cp} records counterparty survival; K_t contains physical inventory and availability; $\mathcal{B}_t$ contains customer commitments; b_t is unrestricted cash; D_t is debt; M_t is posted collateral; and θ_{t-} is the inherited financial position. The factor law generates paths and conditional distributions. A scenario tree discretizes that law.

Three valuation objects remain distinct. A **clean futures reference** represents a continuously settled financial exposure. A **terminal forward** exchanges a price difference at a specified future settlement date. A **physical reservation price** compensates this particular portfolio for committing capacity, credit and liquidity. Shared factors connect their values; settlement and delivery rights distinguish them.

The implemented reference economy has nine factors, six compute curves, two electricity curves, one commercial funding curve, an affine interest-rate model, four physical fleet pools and fifteen dated financial instruments: eleven futures, two funded puts, a transferable power forward and a periodic compute cap. Its numerical parameters are synthetic. The market-law protocol, affine realization and numerical policies are separate software interfaces. User-supplied cases use the same interface. The retained study saves its case and seeds.

2 Delivery conventions and information

2.1 Calendar time, tenor and units

Write T for a calendar delivery date and $x = T - t$ for time to delivery. Financial time is in years; operating quantities use the actual number of GPU-hours or MWh in each interval. A one-year example uses eight delivery intervals of 1,095 hours. No instrument rolls its contractual expiry when the valuation date changes.

A compute service identifies generation, region and service eligibility. Positive compute curves have units of currency per contracted GPU-hour. Electricity curves have units of currency per MWh. Interest and funding rates are annual decimals. A contract multiplier converts each quoted price change into cash.

Hardware-time reservations and useful-work commitments require different quantities. For workload w, configuration j delivers ν_{wj} useful units per GPU-hour, subject to quality, memory, topology and latency eligibility. Only eligible configurations enter the substitution set. The workload simulator tracks remaining work, checkpoint losses, cluster placement and deadlines. Benchmark methodology must fix these conditions before measuring ν_{wj} [R12].

2.2 Observable state

Let $\mathcal{G}_t$ be the information available when a decision is made. A quote or policy is $\mathcal{G}_t$-adapted. Market observations satisfy

$$Y_i = \mathcal{O}_i(t_i, X_{t_i}; \Theta) + \epsilon_i.$$

The observation operator knows whether an input is a dated futures reference, a terminal forward, an arithmetic delivery strip or a physical quote with explicitly supplied contractual adjustments. Indicative references and executable quotes have different source and uncertainty fields. Physical quotes are not used as clean forwards without normalization.

When factors are hidden, the statistical state is the conditional distribution of X_t. The included extended Kalman filter propagates exact factor means and covariances, linearizes each nonlinear observation and uses a Joseph-form covariance update. It reports innovation errors and observation rank. This is a local Gaussian filter for a nonlinear jump model. The held-out operating study uses observed simulated factors; the separate noisy-quote experiment measures filtering performance [R21].

3 Coupled stochastic curves

3.1 Function-valued market state

The `MarketModel` contract provides curves, discounting, conditional transitions and sensitivities; `AffineCurveModel` is its reference realization. Claims and control consume these operations rather than prescribing their stochastic implementation [R25].

For each market a, define the Musiela curve $\mathcal{F}_t^a(x) = F^a(t, t+x)$. Calendar roll contributes the transport operator ∂_x. A generic log-curve representation is

$$df_t^a(x) = \{\partial_x f_t^a(x) + \mu_t^a(x)\}\, dt + \sigma_t^a(x)\, dW_t + dJ_t^a(x), \qquad f_t^a = \log \mathcal{F}_t^a.$$

The drift is tied to the interpretation of F. A clean futures curve satisfies a martingale restriction under its pricing measure, and the implementation enforces that restriction through an affine realization rather than numerically correcting an arbitrary curve process. This follows the forward-curve organization of HJM and its commodity extensions [R15–R18].

3.2 Finite-dimensional realization

Let $K = \operatorname{diag}(\kappa_1, \dots, \kappa_d)$, with $\kappa_i \geq 0$. Under measure $m \in \{P, Q\}$,

$$dX_t = (-KX_t + \eta_m)\, dt + L\, dW_t^m + dJ_t^m.$$

Here J^m is a raw compound-Poisson process with intensity λ_m and Gaussian marks $Z_m \sim N(m_m, V_m)$. The pricing law uses $\eta_Q = 0$. Set $C = LL^\top$. The cumulant of the driving increment is

$$\psi_m(z) = \eta_m^\top z + \frac{1}{2} z^\top C z + \lambda_m \left[\exp\!\left(z^\top m_m + \frac{1}{2} z^\top V_m z \right) - 1 \right].$$

Each curve has a loading vector c_a and maturity shape

$$\ell_a(x) = e^{-Kx} c_a.$$

These shapes are closed under differentiation and time shift: $\partial_x \ell_a = -K\ell_a$. The factor representation is therefore exact for this chosen family. An arbitrary empirical principal-component basis would require an additional closure approximation.

The common factors express level, near-delivery scarcity, long-end slope, generation basis, regional basis, power, interest, funding liquidity and service basis. Shared Brownian covariance and shared jump marks transmit a single event through all affected curves. Fast decay concentrates a shock near delivery. Zero decay supplies a persistent level component. Loadings, not curve labels, determine covariance.

3.3 Positive compute references

For a calibrated initial curve $F_0^a(T) > 0$, define

$$A_a(t, T) = \int_{T-t}^{T} \psi_Q(\ell_a(v))\, dv,$$

$$\boxed{F^a(t,T) = F^a_0(T)\exp[\ell_a(T-t)^\top X_t - A_a(t,T)]}\,.$$

For fixed T, the state drift cancels the derivative of $\ell_a(T-t)$. The remaining exponential compensator is exactly $\psi_Q(\ell_a(T-t))$. Independent-increment conditioning then gives

$$E_Q[F^a(u,T) \mid \mathcal{G}_t] = F^a(t,T), \qquad t \le u \le T.$$

This is a futures-price identity, not a storage relation. No factor of the form spot times $e^{r(T-t)}$ is imposed on compute.

In log-Musiela coordinates, away from initial interpolation knots,

$$df^a_t(x) = \{\partial_x f^a_t(x) - \psi_Q(\ell_a(x))\}\,dt + \ell_a(x)^\top L\,dW^Q_t + \ell_a(x)^\top dJ^Q_t.$$

The cumulant includes the raw-jump compensation. Omitting it would create drift in the clean futures reference.

3.4 GPU, regional and service basis

For local service j and its reference benchmark 0,

$$\log\frac{F^j(t,T)}{F^0(t,T)} = \log\frac{F^j_0(T)}{F^0_0(T)} + [\ell_j(T-t) - \ell_0(T-t)]^\top X_t - [A_j(t,T) - A_0(t,T)].$$

This basis curve inherits its maturity shape from the difference in loadings. Some basis factors have no traded hedge. Inferred local reference curves support valuation and observation filtering; the instrument registry separately states where an executable financial claim exists.

3.5 Normal electricity curves

Electricity can be negative, so its reference uses an additive specification. Let $\bar{j}_Q = \lambda_Q m_Q$. Then

$$F^a(t,T) = F^a_0(T) + \ell_a(T-t)^\top X_t - \int_{T-t}^{T} \ell_a(v)^\top \bar{j}_Q\,dv.$$

The same factors and pricing law apply. The additive jump compensation again makes the fixed-delivery reference a Q-martingale. Negative prices do not require clipping or a log transform [R17, R18].

3.6 Forecasting and pricing laws

Under P, drift and jump law may differ from Q. For a positive reference,

$$E_P[S^a_T \mid \mathcal{G}_t] = F^a(t,T)\exp\left\{\int_0^{T-t}[\psi_P(\ell_a(v)) - \psi_Q(\ell_a(v))]dv\right\}.$$

The futures reference equals expected spot when this risk-premium adjustment vanishes. Statistical estimation uses the physical law; clean derivative valuation uses the pricing law. Market data must identify the pricing-law parameters that affect tradable claims. Sparse observations leave pricing-law directions unidentified; a smooth curve does not resolve that information problem.

The software checks that a Brownian drift change lies in the diffusion span and that the two Gaussian jump laws have compatible support. These conditions exclude singular changes of measure in the implemented constant-coefficient family. Shared jump laws may still generate nonlinear payoff risk that finitely many traded futures cannot span [R19, R20].

4 Interest, discounting and commercial funding

4.1 An affine discount curve on the same state

Let a_r be the rate-loading vector and

$$B(x) = \int_0^x e^{-Kv} a_r \, dv.$$

For a calibrated initial instantaneous forward rate $r_0(t)$, use

$$\phi(t) = r_0(t) + \psi_Q(-B(t)), \qquad r_t = \phi(t) + a_r^\top X_t.$$

The zero-coupon bond is

$$\boxed{P(t,T) = \exp\left[-\int_t^T \phi(u) du - B(T-t)^\top X_t + \int_0^{T-t} \psi_Q(-B(v)) dv \right]}\,.$$

It exactly reproduces $P(0,T) = \exp[-\int_0^T r_0(u) du]$. The endpoint and the integrated factor process are simulated jointly, so pathwise discounting retains covariance with compute prices. The rate model can produce negative rates; it is not constrained by a positive-rate transform [R19].

4.2 Futures versus terminal forwards

A terminally settled forward delivers $S_T - K$ at T. Its zero-value strike is

$$K^{\text{fwd}}(t,T) = \frac{E_Q[e^{-\int_t^T r_u du} S_T \mid \mathcal{G}_t]}{P(t,T)}.$$

For the positive compute curve, write $\tau = T - t$. Then

$$K^{\text{fwd}}(t,T) = F(t,T) e^{\chi(t,T)},$$

$$\chi(t,T) = \int_0^\tau \left[\psi_Q(\ell(v) - B(v)) - \psi_Q(-B(v)) - \psi_Q(\ell(v)) \right] dv.$$

This is the rate–compute convexity adjustment. When the rate loading is zero, $\chi = 0$. The normal-curve implementation uses the corresponding cumulant-gradient identity. Continuously margined futures, terminal forwards and periodic physical invoices consequently have separate settlement operators.

4.3 Commercial funding

A commercial spread is modeled as

$$s_t(x) = s_0(t+x)\exp[\ell_s(x)^\top X_t].$$

It shares the market factors but is not constrained to be a clean traded-futures martingale. Debt contracts determine which reference rate, spread, reset dates and coupon conventions apply. Outstanding fixed-coupon debt keeps its coupon. A change in the funding curve changes new funding economics and refinanced obligations, not an existing fixed coupon.

5 Delivery claims and exercise rights

5.1 Arithmetic delivery aggregation

For delivery quantities q_i at dates T_i, a futures strip reference is

$$\overline{F}_t = \frac{\sum_i q_i F(t, T_i)}{\sum_i q_i}.$$

A fixed-price strip with cash settlement at each delivery date has strike

$$K_t = \frac{\sum_i q_i P(t, T_i) K^{\mathrm{fwd}}(t, T_i)}{\sum_i q_i P(t, T_i)}.$$

These are arithmetic operators on prices, not geometric averages of log-curves. Continuous strips integrate against delivery weights and split quadrature at initial-curve knots. Partitioning a delivery interval and recombining its quantities preserves the original strip value [R16, R18].

Physical service rights add supplier availability, performance, termination, credit and volume terms. Their value belongs in the portfolio operator below. A clean cash-settled cap does not procure a cluster.

5.2 European claims

A claim paying $g(F(e,T))$ at expiry $e \le T$ has value

$$V_t = P(t,e) E_{Q^e}[g(F(e,T)) \mid \mathcal{G}_t].$$

For the diffusion-only model, the reference is lognormal under the expiry-forward measure. The implementation evaluates the Gaussian option formula with the covariance adjustment from the random discount factor. With jumps, the registry uses deterministic discounted affine-transform inversion; independent joint factor and integrated-rate Monte Carlo checks the same payoff. Monte Carlo outputs include standard error, sample count and seed. Put–call parity and the no-rate-convexity limit are tested.

5.3 Flexible exercise

The swing example allows at most one exercise per specified date and at most N exercises overall. Exercise pays $(S_{t_i} - K)^+$ per unit. A backward least-squares recursion estimates continuation values separately for each remaining-rights state. Evaluation freezes those regressions and uses independent paths. The exercise decision uses current observations and remaining rights, never the future realized maximum [R22].

Other payoff functions, including monthly-average compute caps and compute–power margin floors, use the same simulated curves. The common registry brings funded puts, a fixed-strike power forward and a periodic cap into the main curve-control study alongside dated futures. Flexible swing exercise is a separately trained claim policy on the same curve state.

6 Physical assets, collateral and financing

6.1 Operating feasibility

Let y_t denote dispatch and procurement decisions. Physical feasibility requires

$$A_t y_t \le C_t^{\text{usable}}, \qquad y_t \in \mathcal{Y}_t.$$

The set $\mathcal{Y}_t$ includes cluster placement, eligibility, concurrency and indivisibility. Customer commitments consume capacity before discretionary sales. Outages reduce usable physical resources; replacement procurement has explicit prices and limits. Workload and exact placement modules resolve topology when an aggregate pool would conceal fragmentation.

In the curve-control experiment, four compatible fleet pools are operated at block level. A separate placement solver and event-driven workload simulator retain job-level constraints. No financial payoff satisfies a physical obligation by itself.

6.2 Asset appraisal from the curves

The fleet appraisal uses the remaining calendar economic life T_g^{life}, not a horizon that rolls forward each day. Its forward-margin proxy is

$$\widetilde{A}_t^g = \int_t^{T_g^{\text{life}}} P(t,u)\, u_g\, h_g \left[K_g^{\text{fwd}}(t,u) - e_g K_P^{\text{fwd}}(t,u) - o_g\right]^+ du + P(t, T_g^{\text{life}}) R_g.$$

Here h_g converts years to deliverable GPU-hours, u_g is appraisal utilization, e_g is MWh per GPU-hour and o_g is other marginal cost. A fixed initial scale matches the supplied equipment mark. An independent appraisal/liquidity basis captures resale conditions not contained in rental margins.

This forward-margin appraisal differs from the expected value of optimally exercised dispatch options. Lender marks may lag the appraisal and apply eligibility reductions. Liquidation proceeds also include sale discounts and market impact. Thus rental value, economic use value, lender collateral and liquidation cash remain distinguishable.

Cash flows through the operating horizon enter operating wealth. The terminal hardware mark represents cash-generating life after that horizon. This separation prevents counting the same future rental receipts twice.

6.3 Debt and borrowing capacity

For an eligible collateral pool,

$$BB_t = \sum_g \alpha_g A_{g,t}^{\text{eligible}} + \sum_c \beta_c R_{c,t}^{\text{eligible}}.$$

Receivables and equipment require contract-specific eligibility and exclusive pledge allocation. The synthetic curve case sets the receivable advance to zero. Drawable availability respects the facility commitment, drawn balance, borrowing base, draw limits and suspension conditions.

Each debt balance follows actual borrowing and repayment. Principal is not revenue. Interest on the actual post-action balance accrues over the interval and is paid at the next date, including the final interval before terminal repayment. Scheduled term amortization is paid when due. A refinancing assumption is explicit; it is not inferred from a positive asset value.

With collateral value A, debt D, advance rate α, available cure cash b, and sale proceeds χx from selling marked collateral x, compliance requires

$$D - b - \chi x \leq \alpha(A - x).$$

When $\chi > \alpha$, the minimal sale in this linear example is

$$x_{\min} = \frac{(D - b - \alpha A)^+}{\chi - \alpha}.$$

The numerical repair routine includes liquidation impact and physical breakpoints. Selling more pledged equipment need not improve the borrowing-base gap. A cure may instead require cash, new capital or revised financing [R09].

7 Economic assets, lender appraisal and executable exit

A hardware cohort has three values with different uses:

$$A_t^{econ}, \qquad A_t^{lender}, \qquad A_t^{liq}(q).$$

Economic value supports investment and operating decisions. Lender appraisal supports the contractual borrowing base. Liquidation value is the cash obtainable by selling a specified quantity. The first includes operating options; the second is a declared appraisal method; the third includes execution conditions. Adding them would count the same asset more than once.

7.1 Operating, idle and retired states

On decision dates t_n, let s_n denote idle, operating, or the number of mandatory operating intervals remaining. Retirement is absorbing. Admissible actions are run, idle and retire; minimum run duration can remove the latter two. The action also determines the successor operating state $\Gamma(s_n, a_n)$.

For the reference single-cohort problem, the current interval's capacity is sold at its observed opening rental rate. Its interval cash receipt is

$$R_n^{run} = N\,8760\,\Delta t_n\, u\{F^{compute}(t_n, t_n) - eF^{power}(t_n, t_n) - o\} - \text{startup}(s_n).$$

Idle capacity pays standby and any shutdown cost. Retirement receives independently specified resale proceeds, less shutdown cost. These cash flows are known at the decision date. A contract with intraperiod price resets would require additional settlement dates.

Given a declared valuation law M, the operating Bellman equation is

$$A_n(x, s) = \max_{a \in \mathcal{A}(s)} \left\{ R_n(x, s, a) + E_M[D_{n,n+1} A_{n+1}(X_{n+1}, \Gamma(s, a)) \mid X_n = x] \right\}.$$

A selected pricing law Q produces an unencumbered economic value. A P calculation reports expected discounted operating cash flow under the forecasting law; the financed owner's risk preferences enter the separate private-book problem. The distinction avoids charging for debt twice.

Backward least-squares simulation fits action continuations on training paths [R22, R24]. A frozen policy is evaluated on independent paths. A second, pathwise dynamic program sees the entire future and supplies a perfect-information upper comparison. For every evaluated path,

$$PV(\text{admissible policy}) \leq PV(\text{perfect information}).$$

The expected fitted-policy value is a feasible-policy lower bound on the stated finite-grid optimum. The expected perfect-information value is an upper bound. Monte Carlo estimates of those expectations retain sampling errors; the gap combines policy approximation and the value of future information.

7.2 Appraisal and liquidation

The lender's forward-margin appraisal described above is not replaced by the fitted operating value. Its update dates and eligibility rules remain contractual. Executable sale proceeds can be represented by

$$A^{liq}(q) = a\, q\, (1 - h - \zeta q/d)^+ ,$$

where a is the reference resale mark, h an immediate-sale haircut and d market depth. Quantity-dependent liquidation losses feed the cash and collateral ledger. The asset experiment and the financed-book experiment report their quantities and horizons separately so that per-cohort economic value is not compared directly with total-fleet collateral.

8 Portfolio value and reservation pricing

8.1 Private value on the market state

Let $V_t(S_t)$ be the optimized value of the financed operating book. A recursive risk mapping is

$$\mathcal{U}_t(Y) = (1-\omega)E_P[Y \mid \mathcal{G}_t] - \omega ES_{\alpha,t}(-Y).$$

With distributions d_t, injections e_t and admissible controls a_t,

$$V_t(s) = \sup_{a_t \in \mathcal{A}_t(s)} \left\{ d_t - e_t + \mathcal{U}_t[\delta_{t,t+1} V_{t+1}(S_{t+1})] \right\}.$$

Cash flows belong either in the state transition or the immediate objective, not both. Terminal value includes remaining hardware, debt, financial closeout and unfulfilled obligations. Clean derivative valuation uses Q; private risk management uses P and the selected risk mandate. Both use the same curve state [R01–R03, R07, R08].

The finite-control program uses nested conditional risk at its stated decision dates. Its discrete probabilities are generated from the curve transition law. Optional rectangular probability ambiguity changes the risk evaluation, not the clean curve martingale law [R04].

8.2 Contract price

Let $\Phi(s; c, A)$ be the state after accepting obligations c and receiving upfront payment A. The reservation payment is

$$\boxed{A^{\min}(c \mid s) = \inf\{A : V(\Phi(s; c, A)) \geq V(s),\ \Phi(s; c, A) \text{ has a feasible policy}\}.}$$

Receiving cash changes financing feasibility, so the payment enters the state rather than being subtracted from a valuation computed against a fixed cash budget. Opening and closing periodic rates solve the corresponding equation with receipts on their contractual dates. For continuous linear cases, payment is a decision variable and the solver gives a lower certificate; a separate valuation checks the upper payment. Integer cases use bracketed valuation.

A clean strip mark serves as a reference, and its difference from this reservation price is where the delivery, optionality, balance-sheet and risk effects of a particular book appear.

8.3 Marginal capacity and liquidity

Private capacity value is the change in V after adding eligible inventory at a delivery interval. It measures the block's value to this book. The shadow value can jump at placement, funding and contract boundaries.

When V is differentiable, let $m_b = \partial_b V$. For a small contract perturbation with state direction $\dot{s}_c$,

$$\left.\frac{dA^{\min}}{d\epsilon}\right|_0 = -\frac{\nabla V(s)^\top \dot{s}_c}{m_b(s)}.$$

This expresses the capacity and risk burden in units of available cash. A local commercial quote additionally incorporates customer acceptance and cross-configuration substitution. In the one-product exponential-arrival limit, the price is marginal cash cost plus capacity opportunity value divided by m_b, plus the inverse price-elasticity parameter [R05].

9 Curve Greeks and incomplete-market hedging

9.1 From curve perturbations to factors

For positive curve a,

$$\partial_X F^a(t,T) = F^a(t,T)\ell_a(T-t), \qquad \partial^2_{XX} F^a(t,T) = F^a(t,T)\ell_a(T-t)\ell_a(T-t)^\top.$$

A delivery-strip Greek is the same arithmetic weighted sum of its point Greeks. Portfolio Greeks include optimal-policy response, financing and collateral. The research experiment obtains these by common-random-number revaluation of the finite control problem. An unchanged policy would instead measure a frozen-policy Greek.

A functional curve derivative $\delta V / \delta f^a(x)$ projects into factor coordinates as

$$g_k = \sum_a \int \frac{\delta V}{\delta f^a(x)} \ell_{a,k}(x)\, dx.$$

The factor vector is the common risk coordinate for operating contracts, equipment appraisal and financial instruments. Continuous Greeks are local diagnostics; finite jump and funding stresses evaluate nonlinear changes separately [R16, R20].

9.2 Bucketed delivery risk and shape risk

Trader-facing risk uses dated curve buckets rather than only latent factor labels. For each curve, piecewise-linear tent functions $h_i(T)$ partition its delivery grid. Positive compute and funding curves use log-price coordinates; electricity and instantaneous forward rates use absolute-price coordinates. A bucket perturbation changes the initial calibrated curve:

$$f_0^a(T) \mapsto f_0^a(T) + \sum_i z_i h_i(T).$$

The affine compensator and pricing law remain unchanged, so the bumped model still satisfies its pricing identities. Revaluation supplies

$$d_i = \partial_{z_i} V, \qquad G_{ij} = \partial^2_{z_i z_j} V.$$

Central differences use common random numbers; a half-size bump records numerical sensitivity. The reoptimized-book callback includes policy changes, whereas a fixed-claim callback measures clean price risk.

Let J_{ik} be the factor loading at bucket node i. The bridge to factor risk is

$$g^{bucket} = J^\top d.$$

For node-supported linear claims this is exact up to differentiation error. Off-node delivery integrals and nonlinear reoptimized values retain an interpolation residual against directly computed factor Greeks. The software reports that residual.

The projector $P_J = JJ^+$ acts on bucket-shock coordinates. The vector $(I - P_J)^\top d$ identifies sensitivity to curve shapes outside the model's factor span. It is distinct from unhedgeable risk *inside* the factor model. Such off-span shocks are deterministic model-risk tests; the affine simulation does not assign probabilities to them.

9.3 Hedge projection

Let g be the portfolio value delta, H the matrix whose rows are instrument value deltas, and C_h the conditional factor covariance over the hedge horizon. The unconstrained local projection solves

$$\min_q (g + H^\top q)^\top C_h (g + H^\top q).$$

The implementation solves this in covariance-whitened coordinates with a pseudoinverse. It reports the residual, weighted span rank, singular values and the orthogonality error. Duplicate instruments do not create additional hedge dimensions. A basis factor outside the traded span survives even with unrestricted notional.

For a complete delta span, a finite jump can still leave

$$V(X+z) - V(X) + q^\top [H_{\text{price}}(X+z) - H_{\text{price}}(X)] \neq 0.$$

Complete factor rank therefore leaves nonlinear jump risk. The constrained controller uses full conditional revaluation rather than only the quadratic approximation.

9.4 Execution and risk mandate

For positions held over an interval, futures variation settlement is

$$\sum_k q_{k,t-1} m_k [F_k(t) - F_k(t-1)].$$

Execution pays spreads and tiered impact on turnover. Position bounds, turnover caps and impact thresholds are independent inputs. Margin is locked cash, not an economic expense; its funding consequences enter liquidity and borrowing. The curve example sets margin from a factor-covariance stress horizon plus a per-contract floor. It is a synthetic rule, not an exchange schedule.

For conditional future gains G with mean μ, the default hedge mandate solves a joint operating and financial problem containing

$$\rho\{L - (G-\mu)q + Ba\} + TC(q - q^-) + \sum_k f_k IM_k(q) + \sum_k (-\mu_k q_k)^+.$$

Here a contains current dispatch and financing actions. Centering removes rewards for positive forecast trading alpha, while adverse carry remains a cost. Other explicit mandates are centered risk only and total-return optimization. Actual cash settlement follows the claim-specific distribution and trading ledger. Funded-claim mark gains are economic scenario gains, not interim cash receipts.

The additional financial policy induced by a customer contract is

$$\Delta q^*(c \mid \mathcal{B}) = q^*(\mathcal{B} + c) - q^*(\mathcal{B}).$$

This separates contract-driven hedging from any financial allocation the original book would already select.

10 Tradable claims, settlement and execution state

10.1 One claim interface

A tradable claim provides its ex-distribution mark, cash events, factor sensitivities, maturity, multiplier, permitted positions, execution terms and collateral rule. The registry covers futures, European options, fixed-strike transferable forwards, coupon swaps and periodic caps. Every cash-event date must be represented on the control grid.

For futures, inherited inventory generates variation settlement. For a funded claim, changes in its mark remain unrealized until sale; coupon and exercise amounts alone are distributions. If v_n is its ex-distribution mark and d_{n+1} the next distribution, its economic interval gain is

$$G_{n+1} = m\{v_{n+1} + d_{n+1} - v_n\}.$$

Trading from q^- to q exchanges $mv_n(q - q^-)$ in actual cash. Holding inventory receives $mq^- d_{n+1}$, not $mq^-(v_{n+1} - v_n)$. At maturity the ex-distribution mark is zero and the claim's remaining payoff is paid exactly once.

A fixed-strike forward retains its original strike. The periodic cap pays $\sum_n w_n(F_n - K)^+$; an option on an arithmetic average pays $(\sum_n w_n F_n - K)^+$. Those are distinct claims. The registry implements the first and the separate average-payoff utility implements the second.

10.2 Transform pricing of nonlinear inventory

For an option expiring at E on a reference delivered at $T \geq E$, define

$$\mathcal{M}(z) = E_Q\left[D(t,E)\{F(E,T)\}^z \mid X_t = x\right].$$

The discounted moment transform is affine in x with complex argument z [R19]. Fourier inversion under the discount-weighted and asset-weighted distributions gives a call price

$$C = \mathcal{M}(1)P_1(F > K) - K\mathcal{M}(0)P_0(F > K),$$

and a put follows by put–call parity. This retains stochastic-rate and jump dependence. The implementation batches factor states, checks probability/price bounds and compares quadrature resolutions. The no-jump case uses its Gaussian closed form. The transform quadrature requires nonzero projected diffusion; the independent Monte Carlo claim routine also supports degenerate cases.

10.3 Funding and counterparty events

Paid long options require their premium in cash and have no initial margin under the supplied paid-option convention. Short options require an explicit collateral rule. Gross linear margin is used by the path controller; the finite-control adapter also supports bilateral netting and CSA thresholds through its netting-set schema. Initial margin is a segregated asset and returns when inventory is closed.

Positive unsecured exposure at counterparty default receives its specified recovery, while negative amounts remain payable. Default extinguishes affected claims and blocks reopening with that counterparty. The reference path study has one named supplier counterparty and a survival process correlated with funding stress. Clean pricing and reference marks are kept separate from this realized credit settlement convention. Pricing a negotiated credit-adjusted execution spread is an input to the executable quote, not an automatic change to the clean market curve.

The control objective separates expected trading carry, premium cash consumption, margin opportunity charges and debt interest. Instrument-specific liquidity-charge rates are annual decimals multiplied by the actual interval. Cash debt interest is booked separately on actual borrowing. The funded put consumes premium cash at purchase and has no initial margin under the supplied paid-long convention.

10.4 Stochastic execution liquidity

Execution liquidity is a market-access state, not an arbitrageable forward claim. Its reference realization combines an idiosyncratic mean-reverting jump process with shared funding and downside factors:

$$dZ_t = -\kappa_Z Z_t dt + \sigma_Z dW_t^{liq} + dJ_t^{liq}, \qquad \zeta_t = Z_t + \beta_f X_t^{fund} + \beta_d(-X_t^{level})^+.$$

Spreads and margin multipliers increase exponentially with ζ_t, while depth decreases; declared bounds prevent numerical explosions. Above a named stress threshold, executable depth is zero. A factor's name identifies the funding channel, not its array index. This state generates ordinary wrong-way liquidity variation in addition to named interventions [R09].

Position limits, per-period turnover limits and impact-tier thresholds remain distinct instrument inputs. The same sampled liquidity history is used for every policy on a comparison tape. Financial cash already due is settled before the controller responds to a new liquidity state.

11 Identification, estimation and parameter uncertainty

Initial curves, physical transition dynamics, pricing-law option parameters and observation noise are distinct data blocks. A calibration request specifies free parameter coordinates, bounds, scale, fixed factor-gauge anchors and optional priors. Loadings, decay rates, diffusion, physical drift and both jump laws are accessible coordinates. Jump covariance uses Cholesky parameters to preserve positive semidefiniteness.

For factor observations, $X_{n+1} - e^{-K\Delta}X_n$ has conditional characteristic function

$$\varphi_\Delta(u) = \exp\left\{\int_0^\Delta \psi_P(ie^{-Kv}u)dv\right\}.$$

The estimator matches its empirical counterpart at declared frequencies, along with conditional cross-moments that identify persistence. Market observations enter as normalized repricing errors. These blocks form a bounded weighted least-squares problem [R26]. The supplied weights are part of the estimator specification and are calibrated for this design rather than chosen as a fully efficient GMM weighting matrix.

When only prices are observed, a separate latent-history estimator maximizes a Gaussian quasi-likelihood using exact jump-inclusive first and second transition moments and nonlinear observation linearization. This preserves the distinction between observed factors in a synthetic recovery experiment and hidden factors in an actual quote history [R21].

Identification is measured from the scaled **data-only** Jacobian. Prior penalties leave its rank unchanged, so a singular direction stays unidentified even when regularization produces a unique optimizer. Calibration outputs include singular values, active bounds, solver status, source hashes and fitted-model identity.

Complete independent paths are resampled for the synthetic cluster bootstrap. Quotes are perturbed at their stated measurement errors. Every draw is refitted; failed fits are reported. Predictive uncertainty separates

$$\operatorname{Var}(Y \mid \mathcal{D}) = E_\Theta[\operatorname{Var}(Y \mid \Theta, \mathcal{D})] + \operatorname{Var}_\Theta(E[Y \mid \Theta, \mathcal{D}]).$$

The first component is process variation; the second is variation across parameter draws. A real single dependent history requires a block-resampling design rather than treating successive dates as independent clusters. The executable synthetic recovery study fixes nuisance coordinates explicitly and validates held-out derivative prices. It provides software and identification evidence, not a calibration to actual GPU transactions.

12 Numerical architecture and evidence

12.1 Simulation and control

The affine factor transition is sampled from its exact endpoint law. Brownian endpoint and integrated-factor noise are sampled jointly; compound-Poisson event counts, event ages and marks are explicit. Smooth cumulant and covariance integrals use fixed Gaussian quadrature. There is no Euler time-step bias in factor endpoints for the stated constant-coefficient model. Operational and settlement discretization remains a separate numerical choice.

The path controller fits quadratic factor-state continuation components on independent training paths. These components mark a specified future physical plan. At each decision date, a conditional LP jointly chooses current discretionary dispatch, borrowing, repayment, dated futures and funded claims, subject to common cash constraints. It is receding-horizon control with a fitted continuation, not a solved global Bellman function. Replacing the continuation method leaves the curve law, claims, balance sheet and settlement operators unchanged [R11, R22].

Incoming variation settlement, accrued interest, scheduled payments and inherited-margin requirements are handled before rebalancing. Mandatory financing repair may sell pledged capacity. New trades, margin and discretionary delivery costs must then fit current cash. Closing receipts arrive after prefunding. This chronology makes financing failure a path event rather than a penalty added at maturity.

12.2 Finite control programs

The adapter samples the same factor transition at each observable-history node and maps its curve values into the exact LP/MILP backend. It retains the existing book, offered contract, model identity, factor states and date grid. Instrument positions and physical decisions are node-adapted. The pricing study fixes the horizon and varies branching and random seed.

The compiled model records objective, dual bound, gap, residuals and integrality separately from any secondary turnover tie-break. Finite-support expected shortfall is reported with conditional tail mass and worst-child saturation, which bound the precision available from the conditional support [R10, R13, R14].

12.3 Reproducibility contract

A complete case includes the curve model, fleet, debt terms, existing commitments, offered contract, instrument definitions and assumptions. Its canonical JSON has a SHA-256 identity. Simulation tapes retain model identity, factors, integrated rates and physical events. Policy artifacts retain the training law, continuation fit, conditional seed and mandate. Training and evaluation use different seeds; policy comparisons share evaluation shocks.

The research program checks martingales, initial discount fit, futures–forward convexity, strip partitioning, option parity, analytic Greeks, filter observability, jump hedge residuals, conditional tail support, independent cash reconstruction and replay. Named stresses modify factors or explicit execution conditions. They are experiments, not assigned occurrence probabilities.

The dependencies are

$$\boxed{\text{coupled curves} \;\longrightarrow\; \text{claims and factor exposures} \;\longrightarrow\; \text{financed portfolio value and control.}}$$

Policies, exercise algorithms and scenario approximations consume the common curve state.

13 Reference map

[R01–R14] ground risk, control, operating pricing, funding, computation and measurement. [R15–R18] ground term-structure organization, commodity-curve realizations and delivery functionals. [R19] grounds affine jump transforms, [R20] incomplete-market hedging, [R21] filtering, [R22] least-squares exercise and [R23] curve-functional derivatives, [R24] operating switching, [R25] finite-dimensional curve realizations and [R26] empirical characteristic-function estimation. Full bibliographic records, links and the precise use of each source are in `REFERENCES.md` and the companion *Model choices and grounding* document.